Hybrid Grid Multiple-Model Estimation with Application to Maneuvering Target Tracking

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***Abstract*— This paper considers the problem of state estimation for a hybrid system with Markovian switching parameters in a continuous space. We propose a hybrid grid multiple model (HGMM) estimator whose model set is a combination of a fixed coarse grid and an adaptive fine grid. We also present two model-set design methods by moment matching, and apply them to practical HGMM algorithms. Simulation results show their cost-effectiveness for state estimation in maneuvering target tracking.**

Keywords: Multiple model, model-set design, maneuvering target tracking.

I. INTRODUCTION

A hybrid system involves two types of components: the *base state* which varies continuously and the *mode* or *modal state* which may jump only. For the estimation problem of hybrid systems, the multiple model (MM) approach is the state-of-the-art solution [1]. Here, a set of models is designed to cover the possible system behavior patterns or structures and the overall output is obtained by a certain combination of the outputs based on each individual model. This approach has a parallel structure and is cost-effective and robust. Due to its unique power, MM estimation has achieved great success in many areas, especially in target tracking, fault detection and isolation.

The mode or modal state is usually treated as a discrete random variable. In some applications, however, the mode space (i.e., the set of possible values of the mode) is a continuous region, and the common practice in MM estimation is to choose or design a finite set of models to approximate this mode space (see, e.g., [2]–[6]). Although the problem of efficient model-set design for MM estimation is still open, some general model-set design methods have been proposed in [6], [7]. The model set so designed are primarily used for fixed-structure MM (FSMM), which uses a fixed set of models at all times. However, when applying the FSMM to hybrid estimation, we sometimes encounter two problems: First, the chosen model set may not cover the full range of the mode, and the truth may lie between adjacent models. Second, even if the chosen model set is large enough to cover the full range, use of all those models does not necessarily guarantee performance improvement, not to mention the prohibitively large computational cost. It was demonstrated in [8] that use of too many models may be as bad as use of too few models.

To overcome defects of FSMM estimation and increase cost-effectiveness, MM estimation with variable structure (VSMM) was proposed in [9], [10] and [8], in which the model set is time varying. In particular, model set adaptation (MSA), which aims to determine the model set at each time for the MM estimation, is one of the core functional components of VSMM. One VSMM algorithm differs from another primarily with respect to how the model set adapts [11].

Several VSMM algorithms with different schemes of MSA have been proposed over the past years, and their four representatives are: model-group-switching (MGS) [12], [13], likely-mode-set (LMS) [14], expected-mode-augmentation (EMA) [15] and adaptive grid (AG) [8]. MGS employs a particular group to run at any given time chosen by a decision. LMS uses a set of models that are not unlikely to match the system mode in effect at any time. EMA uses the model set that is the original model set augmented by an expected model (set) at any time. AG sets up a coarse grid initially, and then adjusts the grid recursively according to an adaptation rule possibly based on the current estimate, mode probabilities and measurement residuals. In particular, EMA makes it possible to cover a large continuous mode space with a relatively small number of models at a given accuracy level. Compared with the FSMM algorithms, these VSMM algorithms have been shown to have a considerable improvement in cost-effectiveness.

It has been theoretically justified that adding a new model set which is close to the true model will improve the performance of MM estimation [15]. Stimulated by this idea, we have developed an MM estimation algorithm, called the hybrid grid multiple model (HGMM) estimator [16]. Unlike the AGMM, HGMM uses a hybrid grid which consists of a *fixed coarse* grid and an *adaptive fine* one and this hybrid grid tries to cover a large continuous mode space with a relatively small number of models at a given accuracy level. This scheme is particularly advantageous when the mode space

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is continuous and large and the mode involves jumps of a small or medium magnitude.

On the other hand, the performance of MM estimation depends largely on the model set used. Thus the goal of this paper is to propose model-set design methods by moment matching and to develop cost-effective HGMM algorithms that combine with these methods, as a strengthening of [16]. Via simulation in the context of maneuvering target tracking, the performance of the developed HGMM algorithms with different model sets is evaluated.

This paper is organized as follows. A general description of HGMM is presented in Section II. In Section III, two general model-set design methods are presented. Practical HGMM algorithms for maneuvering target tracking are developed in Section IV. Simulation and discussion are given in Section V. Section VI provides conclusions.

II. HGMM Estimation

A. Description of HG

In most MM estimation for the hybrid system whose parameters belong to a continuous space, a general way for the model-set design is to quantize the mode space with a suitable (not too large) number of models. FSMM estimation for the hybrid system uses this set of models. It can not produce an accurate value of the mode when the true mode lies between adjacent models, although the true mode may be approximated by combining these models. On the other hand, [15] has justified theoretically that augmenting a *good* model (or set) can certainly improve the performance of MM estimation. If the true mode is known and added to the model set, [17] showed that MM estimator will converge to the true mode with probability 1, and the optimal state estimate can also be obtained. But it is unrealistic because of the uncertainty of the true mode. Practically, a common way is to add the optimal mode estimate, which is statistically close to the true mode (see, e.g., [18], [19], [20]). In principle, the optimal mode estimate can be an estimate of the true mode under some optimality criterion. For instance, in the sense of minimum mean-square error (MMSE) and maximum likelihood (ML), mode estimate $\hat{s}_k$ can be expressed as

$$\hat{s}_k = \begin{cases} E[m|m \in \mathcal{M}_k, \mathcal{M}^{k-1}, z^k] & \text{MMSE} \\ \arg\max_m f(z_k|m \in \mathcal{M}_k, \mathcal{M}^{k-1}, z^{k-1}) & \text{ML} \end{cases} \quad (1)$$

where $\hat{s}_k$ is the estimate of the true mode at time k, m is the model in the model set $\mathcal{M}_k$, z^k is the measurement sequence through time k, and $\mathcal{M}^{k-1}$ is the model-set sequence through time $k-1$.

Previous work only employs the estimate of the true mode in this type of MM estimation. Here we argue that mode estimation error should also be incorporated into model inference. Based on this idea, our HG scheme acts as follows. The model set in effect at the current time consists of two types of model subsets: fixed coarse and adaptive fine. The coarse subset is obtained by quantizing (uniformly or nonuniformly) the mode space crudely, that is, the spacing between the quantization levels is large, and it is fixed at all times. Since the coarse grid stays unchanged, its number of models should not be large. The fine subset is designed from the region surrounding the optimal estimate of the true mode according to the mode estimation error, and its quantization is much finer than the coarse subset. The true mode may jump, so the fine subset is adaptive and time-varying.

TABLE I
One Cycle of HGMM algorithm

S1: Obtain the mode estimate $\hat{s}_k$ and covariance C_s based on $\{M_{k-1}, \mu_*\}$;
S2: Design the fine subset A_k using the mode estimate $\hat{s}_k$ and its covariance C_s;
S3: Run VSIMM$[M_k, M_{k-1}]$, where $M_k = M \cup A_k$.

Using the hybrid grid has the following advantages:

1) The coarse grid provides a robust scheme to handle abrupt jumps of the system mode and directs the placement of the fine grid. The fine grid can be adapted in a relatively small and better unit, which intuitively makes the mode and base state estimate more accurate.

2) The HG, in which the mode estimation error as well as the mode estimate is incorporated into the model inference, can be viewed as a generalization of the EMA. The HG scheme is suitable for estimation for systems whose mode involves jumps of different magnitudes.

Of course, the main disadvantage of this scheme is obvious—it can be used only for problems in which the mode space is continuous.

B. HGMM Algorithm

Denote by A_k the adaptive fine subset and by M the fixed coarse subset. Then the model set $\mathcal{M}_k$ running at time k equals $M \cup A_k$. In our HGMM algorithm, the model set evolves as

$$\mathcal{M}_{k-1} = M \cup A_{k-1} \ \rightarrow \ \mathcal{M}_k = M \cup A_k$$

That is, the model set $\mathcal{M}_k$ in effect at time k is equal to $\mathcal{M}_{k-1}$ with the fine subset A_{k-1} at time $k-1$ replaced by A_k at time k.

The HGMM algorithm is described in Table I. The functional module VSIMM$[\mathcal{M}_k, \mathcal{M}_{k-1}]$ involved in the algorithm denotes the recursion for variable structure interacting multiple model (IMM) estimation that uses model set $\mathcal{M}_k$ and $\mathcal{M}_{k-1}$ at time k and $k-1$. It has been developed and details can be found in [21], [22].

The probabilities μ_* of models at time k can be $\mu_{k|t}$, i.e., the predicted probabilities if $t < k$, the updated probabilities if $t = k$, or the smoothed probabilities if $t > k$. In general, a larger t leads to a more accuracy μ_*, but requires more computation.

C. Determination of The Fine Subset

As described before, the coarse subset M is fixed, and it can be chosen or obtained by some model-set design approaches, such as those proposed in Section III. Next, we explain how to determine the fine subset A_k, which is Step 2 in the HGMM algorithm.

In this paper, $\hat{s}_k$ is chosen under the MMSE criterion. $\mathcal{M}_k$ contains A_k, which is determined by $\hat{s}_k$. The mode estimate $\hat{s}_k$ is obtained as

$$\hat{s}_k \triangleq \bar{s}_k = \sum_{m^{(j)} \in \mathcal{M}_{k-1}} \bar{m}^{(j)} \mu_*^{(j)} \tag{2}$$

and its corresponding covariance is

$$\begin{aligned} \boldsymbol{C}_s = \sum_{m^{(j)} \in \mathcal{M}_{k-1}} \mu_*^{(j)} \Big[\text{Cov}\left(m^{(j)} \right) \\ + \left(\hat{s}_k - \bar{m}^{(j)} \right) \left(\hat{s}_k - \bar{m}^{(j)} \right)' \Big] \end{aligned} \tag{3}$$

where $\mu_*^{(j)} = \mu_{k|k}^{(j)}$, $\mu_{k|k-1}^{(j)}$ or something similar. Here, $\mu_{k|k}^{(j)} = P\{s_k = m^{(j)} | m^{(j)} \in \mathcal{M}_{k-1}, z^k\}$ and $\mu_{k|k-1}^{(j)} = P\{s_k = m^{(j)} | m^{(j)} \in \mathcal{M}_{k-1}, z^{k-1}\}$ denote the updated and predicted probabilities of model $m^{(j)}$ being the correct one, and $\{\bar{m}^{(j)}, \text{Cov}\left(m^{(j)}\right)\}$ is the parameter that characterizes model $m^{(j)}$.

Given the expectation and covariance of the parameter that characterizes the true mode, [16] proposed a method for the design of the fine subset which is obtained by quantizing the confidence region of the true mode, and presented two approaches which are simple and easy to implement. In order to make HGMM more cost-effective with less models, here we present optimal design methods for the fine subset which are detailed in the next section.

III. Model Set Design Methods Using Moment Matching

Like the formulation of model-set design in [6], the designed model m and the true mode s are viewed as random variables. When some moments or the distribution of the true mode is known, we can utilize this information to design the model set for MM estimation. In this section, we want to design a model set $\mathbb{M} = \{p_i, m_i\}$ $(i = 1, ..., r)$, where r is the number of models, and p_i corresponds to the probability of model m_i. Here, only the mean and covariance matching is considered. Assume the true mode $s \sim f\left(\bar{s}, \boldsymbol{C}_s\right)$, where $\bar{s}$ and $\boldsymbol{C}_s$ denote the first two moments of s, and the designed model $m_i \sim g\left(\bar{m}_i, C_i\right)$, where $\bar{m}_i$ and C_i are the mean and covariance of model m_i. Then, this design needs to satisfy the following conditions under the moment matching criterion:

$$\begin{aligned} \sum_{i=1}^{r} p_i = 1, \ \bar{s} = \sum_{i=1}^{r} p_i \bar{m}_i \\ \boldsymbol{C}_s = \sum_{i=1}^{r} p_i \left[C_i + \left(\bar{m}_i - \bar{s}\right)\left(\bar{m}_i - \bar{s}\right)' \right] \end{aligned} \tag{4}$$

Note that the conditions are the same as those in [6] except that the deterministic model m_i there is replaced by the local mean $\bar{m}_i$ (since m_i is now considered random) and the covariance C_i of model m_i is considered here.

A. Minor Model-Set Design

If we set $C_i = (1-\beta)\, \boldsymbol{C}_s$, where $0 \le \beta \le 1$, then Equation (4) can be written as

$$\sum_i p_i = 1, \ \sum_i p_i \bar{m}_i = \bar{s}, \ \sum_i p_i (\bar{m}_i - \bar{s})(\bar{m}_i - \bar{s})' = \beta \boldsymbol{C}_s \tag{5}$$

By transformation $\bar{m}_i = B \bar{\mathsf{m}}_i + \bar{s}$, where $\beta \boldsymbol{C}_s = BB'$, the design of $\{p_i, \bar{m}_i\}$ can be converted to the standard design of $\{p_i, \bar{\mathsf{m}}_i\}$ with

$$\sum_i p_i = 1, \ \sum_i p_i \bar{\mathsf{m}}_i = 0, \ \sum_i p_i (\bar{\mathsf{m}}_i - \bar{s})(\bar{\mathsf{m}}_i - \bar{s})' = I \tag{6}$$

The following theorem provides a solution to the standard design, and its proof can be found in [6].

Theorem 1 (Minor-Set Design): The design $\{p_i, \bar{\mathsf{m}}_i\}$ with $0 \le p_0 \le 1$

$$\begin{aligned} & p_0^1 = p_0, \ p_1^1 = p_2^1 = (1-p_0)/2 \\ & \bar{\mathsf{m}}_0^1 = \mathbf{0}, \ \bar{\mathsf{m}}_1^1 = (1-p_0)^{-1/2}, \ \bar{\mathsf{m}}_2^1 = -(1-p_0)^{-1/2} \\ & \quad \vdots \\ & p_0^j = p_0, \ p_i^j = p_i^{j-1}/2, \ i = 1, ..., j, \ p_{j+1}^j = (1-p_0)/2 \\ & \bar{\mathsf{m}}_0^j = \mathbf{0}, \ \bar{\mathsf{m}}_i^j = \left[\left(\bar{\mathsf{m}}_i^{j-1} \right)' \ \ (1-p_0)^{-1/2} \right]', \ i = 1, ..., j, \\ & \bar{\mathsf{m}}_{j+1}^j = \left[\mathbf{0} \ \ -(1-p_0)^{-1/2} \right]' \end{aligned}$$

satisfies Equation (6).

Remark: To match the mean $\bar{s}$ and covariance $\boldsymbol{C}_s$ of the true mode s, the minimum number of models needed for m is rank$(\boldsymbol{C}_s) + 1$ [6]. In the minor model set design, the number of models designed is rank$(\boldsymbol{C}_s) + 2$.

B. Partitioned Moment Matching Design

The idea of partitioned moment matching method is that we partition the mode space $\mathbb{S}$ into a set of $\mathcal{S}_i$ which are disjoint and exhaustive, that is,

$$\mathcal{S}_i \cap \mathcal{S}_j = \emptyset \text{ for } \forall i \neq j, \quad \textstyle\bigcup_i \mathcal{S}_i = \mathbb{S}$$

Assume that the pdf $f(s)$ of the true mode s is known. For $\mathcal{S}_i$, define

$$\begin{aligned} p_i &\triangleq P\left\{s \in \mathcal{S}_i\right\} = \int_{\mathcal{S}_i} f(s)\, ds \\ \bar{m}_i &\triangleq \int_{\mathcal{S}_i} s f\left(s | s \in \mathcal{S}_i\right) ds = \frac{1}{p_i} \int_{\mathcal{S}_i} s f(s)\, ds \\ C_i &\triangleq \int_{\mathcal{S}_i} \left(s - \bar{m}_i\right)\left(s - \bar{m}_i\right)' f\left(s | s \in \mathcal{S}_i\right) ds \\ &= \frac{1}{p_i} \int_{\mathcal{S}_i} \left(s - \bar{m}_i\right)\left(s - \bar{m}_i\right)' f(s)\, ds \end{aligned}$$

Then we have the following theorem.

Theorem 2: If m has the mixture pdf $\sum_i g_i(m; \bar{m}_i, C_i) p_i$, where $g_i(\bar{m}_i, C_i)$ denotes a distribution with mean $\bar{m}_i$ and covariance C_i, and p_i satisfies

$$\sum_i p_i = 1, \quad p_i > 0$$

then the mean $\bar{m}$ and the covariance $\boldsymbol{C}_m$ of m are the same as those of $s \sim f(\bar{s}, \boldsymbol{C}_s)$.

Proof: The mean of s is

$$\bar{s} = \sum_i P\{s \in \mathcal{S}_i\} \int_{\mathcal{S}_i} sf(s|s \in \mathcal{S}_i)\, ds = \sum_i p_i \bar{m}_i = \bar{m}$$

The covariance of s is

$$\begin{aligned}
\boldsymbol{C}_s &= \sum_i P\{s \in \mathcal{S}_i\} \int_{\mathcal{S}_i} (s-\bar{s})(s-\bar{s})' f(s|s \in \mathcal{S}_i)\, ds \\
&= \sum_i p_i \left[C_i + (\bar{m}_i - \bar{s})(\bar{m}_i - \bar{s})'\right] \\
&= \sum_i p_i \left[C_i + (\bar{m}_i - \bar{m})(\bar{m}_i - \bar{m})'\right] \\
&= E\left[(m-\bar{m})(m-\bar{m})'\right] = \boldsymbol{C}_m
\end{aligned}$$

Therefore, this design is moment matching. ■

Here, we consider a special partitioning for the mode space $\mathbb{S}$ with the assumption that mode s has a Gaussian probability density function (pdf) $f(s) = \mathcal{N}(s; \bar{s}, \boldsymbol{C}_s)$[1],

$$\mathcal{S}_i : d_{i-1} \le \alpha' s < d_i \tag{7}$$

where α is a vector with the same dimension as mode s, and d_{i-1} and d_i are both scalars. The probability p_i, the mean $\bar{m}_i$ and the covariance C_i of m_i can be obtained from the following theorem.

Theorem 3: In $\mathcal{S}_i$ defined by Formula (7), $\bar{m}_i$ and C_i are the mean and covariance of model m_i, and p_i is the corresponding model probability. Let

$$K = \boldsymbol{C}_s \alpha \left(\alpha' \boldsymbol{C}_s \alpha\right)^{-1}$$

Then

$$\begin{aligned}
p_i &= \int_{d_{i-1}}^{d_i} \mathcal{N}(s; \alpha'\bar{s}, \alpha' \boldsymbol{C}_s \alpha) ds \\
\bar{m}_i &= \bar{s} + K(E - \alpha'\bar{s}) \\
C_i &= \boldsymbol{C}_s - K(\alpha' \boldsymbol{C}_s \alpha - \Sigma) K'
\end{aligned}$$

where

$$\begin{aligned}
E = &-\frac{\alpha' \boldsymbol{C}_s \alpha}{p_i}[\mathcal{N}(d_i; \alpha'\bar{s}, \alpha' \boldsymbol{C}_s \alpha) \\
&- \mathcal{N}(d_{i-1}; \alpha'\bar{s}, \alpha' \boldsymbol{C}_s \alpha)] + \alpha'\bar{s} \\
\Sigma = &-\frac{\alpha' \boldsymbol{C}_s \alpha}{p_i}[(d_i + \alpha'\bar{s}) \mathcal{N}(d_i; \alpha'\bar{s}, \alpha' \boldsymbol{C}_s \alpha) \\
&- (d_{i-1} + \alpha'\bar{s}) \mathcal{N}(d_{i-1}; \alpha'\bar{s}, \alpha' \boldsymbol{C}_s \alpha)] \\
&+ \alpha'\bar{s}\bar{s}'\alpha + \alpha' \boldsymbol{C}_s \alpha - EE'
\end{aligned}$$

For a proof, see [23].

Remark: Suppose a random variable s has pdf $f(s; \bar{s}, \boldsymbol{C}_s)$[2], where $\bar{s}$ and $\boldsymbol{C}_s$ are the mean and covariance of s, and we want to approximate it with a Gaussian pdf $f'(s) = \mathcal{N}(s; \bar{s}^*, \boldsymbol{C}_s^*)$. Then, the choice $\bar{s}^* = \bar{s}$, $\boldsymbol{C}_s^* = \boldsymbol{C}_s$ minimizes the Kullback-Leibler (KL) discrimination $D_{\text{KL}}(f(s), f'(s))$ [24]; that is, $\arg\min_{[\bar{s}^*, \boldsymbol{C}_s^*]} D_{\text{KL}}(f(s; \bar{s}, \boldsymbol{C}_s), \mathcal{N}(s; \bar{s}^*, \boldsymbol{C}_s^*))$ turns out to be equal to $(\bar{s}, \boldsymbol{C}_s)$.

Thus, given the first two moments of mode s, we can use a Gaussian pdf to approximate it and use *Theorem 3* to complete the model-set design.

[1] $\mathcal{N}(s; \bar{s}, \boldsymbol{C}_s)$ denotes the Gaussian distribution

$$\mathcal{N}(s; \bar{s}, \boldsymbol{C}_s) = |2\pi \boldsymbol{C}_s|^{-1/2} \exp\left[-\frac{1}{2}(s-\bar{s})' \boldsymbol{C}_s^{-1}(s-\bar{s})\right]$$

where $\bar{s}$ and $\boldsymbol{C}_s$ are the mean and covariance of s.

[2] $f(s; \bar{s}, \boldsymbol{C}_s)$ denotes a pdf (not necessarily Gaussian) of s with mean $\bar{s}$ and covariance $\boldsymbol{C}_s$.

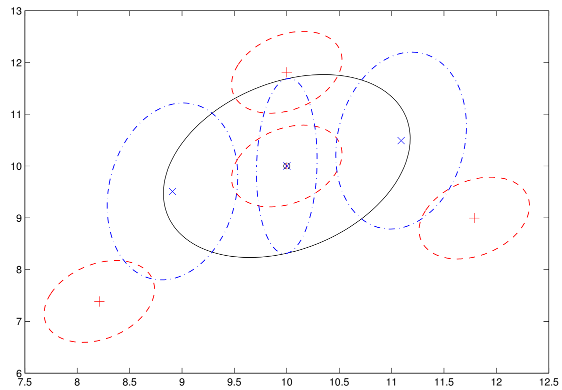


Fig. 1. Illustration of model set design using moment matching methods

C. Example

Assume that $f(s) = \mathcal{N}(s; \bar{s}, \boldsymbol{C}_s)$, where $\bar{s} = \begin{bmatrix} \bar{\mathsf{s}}_1 \\ \bar{\mathsf{s}}_2 \end{bmatrix}$, $\boldsymbol{C}_s = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$, and $-1 < \rho < 1$. It can be shown that

$$\sqrt{\boldsymbol{C}_s} = \begin{bmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sigma_2\sqrt{1-\rho^2} \end{bmatrix}$$

In this example, $\bar{\mathsf{s}}_1 = \bar{\mathsf{s}}_2 = 10$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, and $\rho = 0.3$. Fig. 1 illustrates the model-set designs using the minor model-set method with the factor $\beta = 0.8$ and the partitioned moment matching method. The partitioned moment matching method uses planes to split the mode space into N members with equal probability, where $\alpha = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $p_i = 1/N$ and $N = 3$. In Fig. 1, a circle indicates the estimate of the true model, a solid (black) line indicates the corresponding 1-σ error ellipse, a cross indicates the mean of the partitioned model set, a dash-dot (blue) line indicates the corresponding 1-σ error ellipse, a plus sign indicates the mean of the minor model set, and a dash (red) line indicates the corresponding 1-σ error ellipse.

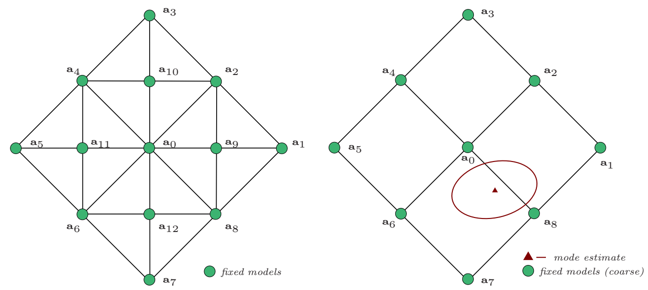
IV. HGMM for Maneuvering Target Tracking

In this section, we use the HGMM estimator to track a maneuvering target.

Consider the following system model of the target

$$\begin{aligned}
\mathbf{x}_{k+1} &= F\mathbf{x}_k + G(\mathbf{a}_k + w_k) \\
\mathbf{z}_k &= H\mathbf{x}_k + v_k
\end{aligned}$$

where $\mathbf{x} = (x, \dot{x}, y, \dot{y})'$ is the state vector, $\mathbf{z}$ is the measurement vector, and $\boldsymbol{a} = (a_x, a_y)'$ is the acceleration. $w \sim \mathcal{N}(\mathbf{0}, Q_{\boldsymbol{a}})$



(a) 13-model set in FSMM (b) fixed subset and mode estimate in HGMM

Fig. 2. Digraph representation of model set designs

and $v \sim \mathcal{N}(\mathbf{0}, R)$ are mode-dependent white Gaussian process and measurement noises, respectively, mutually independent, and the initial state $\mathbf{x}_0 \sim (\bar{\mathbf{x}}_0, P_0)$ is also independent of w and v. $F = \text{diag}[F_2, F_2]$, and $G = \text{diag}[G_2, G_2]$ with

$$F_2 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, G_2 = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

In addition, assume that the values of $\boldsymbol{a}$ are quantized from the continuous acceleration region A^c

$$A^c = \{(a_x, a_y) : |a_x| + |a_y| \leq a_{\max}\}$$

and they are governed by a Markov process, where $a_{\max}$ is a target-type-dependent constant—the maximum acceleration in any direction.

The FSMM, to be compared with our proposed HGMM, is based on a 13-model set $M_{13} = \{\mathbf{a}_j, j = 0, ..., 12\}$, which is uniformly quantized from A^c. The model set for FSMM remains unchanged and runs at every time. The corresponding fixed grid is illustrated in Fig. 2(a).

The HGMM chooses $M = \{\mathbf{a}_j, i = 0, ..., 8\}$ as the coarse grid, as illustrated in Fig. 2(b). Before designing the fine subset, we must obtain the mode estimate $(\bar{a}, C_a)$ which is statistically closest to the true mode:

$$\bar{a} = \sum_{a_j \in \mathcal{M}_{k-1}} \bar{a}_j \mu_*^{(j)} \tag{8}$$

$$C_a = \sum_{a_j \in \mathcal{M}_{k-1}} \left[Q_j + (\bar{a} - \bar{a}_j)(\bar{a} - \bar{a}_j)'\right] \mu_*^{(j)} \tag{9}$$

where $\mu_*^{(j)} \triangleq \mu_{k|k-1}^{(j)} = \sum_i \pi_{ij} \mu_{k-1}^{(i)}$ with model transition probability $\pi_{ij} = P\{s_k = a^{(j)} | s_{k-1} = a^{(i)}\}$. Fig. 2(b) illustrates the expected location and 1-σ error ellipse of the true mode. It is obvious that the covariance of the mode achieved by (9) is conservative (or pessimistic), so we approximate the distribution with a series of distributions of fine models. The fine model set design is based on the two methods of Section III , and the parameters are set as follows:

1) Minor Model Set: $\beta = 0.8$, the number of fine models is rank$(C_a) + 2$ and is equal to 4.
2) Partitioned Moment Matching: partition factor[3] $\alpha = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and the number of fine models $N = 3$.

Note that the scatter degree of the fine models depends on the size of the error ellipse: the larger (smaller) the size is, the less (more) concentrated the fine models are. So, the generation or design of the fine models is an adaptive process.

V. Simulation and Discussion

A. Test Scenarios

We compare these design methods under the following scenarios.

1) Scenario 1: This scenario for a highly maneuvering target is shown in Fig. 3 with $a_{\max} = 80\text{m/s}^2$. The target starts at a constant velocity of 426m/s for 30s and then performs a 7g left turn. The new course is maintained for 30s. A 6g right turn is then performed while the throttle is reduced and the aircraft decreases altitude. After 30s, another 6g turn is performed and the full throttle is commanded. After about another 30s, a 7g right turn is performed and then the straight and level and nonaccelerating flight is maintained until the completion of the trajectory. This benchmark trajectory of a highly maneuvering target was proposed in [25] and adopted in [26]. For simplicity, the altitude of the target is ignored, and the target is only considered in the plane. Assume that the sensor measurement sequence is corrupted by noise $v \sim \mathcal{N}(\mathbf{0}, R)$ with $R = 1250I$. The sampling period $T = 1$s. The unknown acceleration sequence is covered by a set of 13 discrete models

$\bar{\mathbf{a}}_0 = [0, 0]'$	$\bar{\mathbf{a}}_1 = [80, 0]'$	$\bar{\mathbf{a}}_2 = [40, 40]'$
$\bar{\mathbf{a}}_3 = [0, 80]'$	$\bar{\mathbf{a}}_4 = [-40, 40]'$	$\bar{\mathbf{a}}_5 = [-80, 0]'$
$\bar{\mathbf{a}}_6 = [-40, -40]'$	$\bar{\mathbf{a}}_7 = [0, -80]'$	$\bar{\mathbf{a}}_8 = [40, -40]'$
$\bar{\mathbf{a}}_9 = [40, 0]'$	$\bar{\mathbf{a}}_{10} = [0, 40]'$	$\bar{\mathbf{a}}_{11} = [-40, 0]'$
$\bar{\mathbf{a}}_{12} = [0, -40]'$		

and their corresponding acceleration error covariance are

$$Q^0 = I, \; Q^i = 2^2 I, \; i \neq 0 \tag{10}$$

where superscript i denotes quantities pertaining to a_i.

2) Scenario 2: In order to provide a performance comparison as fairly as possible, the algorithms are also tested under a random scenario, proposed in [13], since generally the evaluation of MM algorithm depends to a large extent on the scenario used.

In the random scenario, the acceleration vector $\boldsymbol{a}(t) = a(t)\angle\theta(t)$ is a 2-dimensional semi-Markov process. It satisfies: (a) the sojourn time τ conditioned on a_k is a truncated Gaussian ($\tau > 0$) with mean $\bar{\tau}$ and covariance σ_τ^2; (b) the magnitude a_{k+1} of the acceleration vector is assumed to equal to zero or $a_{\max}$ with probabilities P_0 and P_M, respectively, and uniformly distributed over the values in between; (c) the phase angle θ_{k+1} of the acceleration vector is uniform over $[0, 2\pi]$ if $a_k = 0$ and is Gaussian with mean θ_k and covariance σ_θ^2 if $a_k \neq 0$.

[3] α is a user-defined value. It can be time invariant or varying (adaptive).

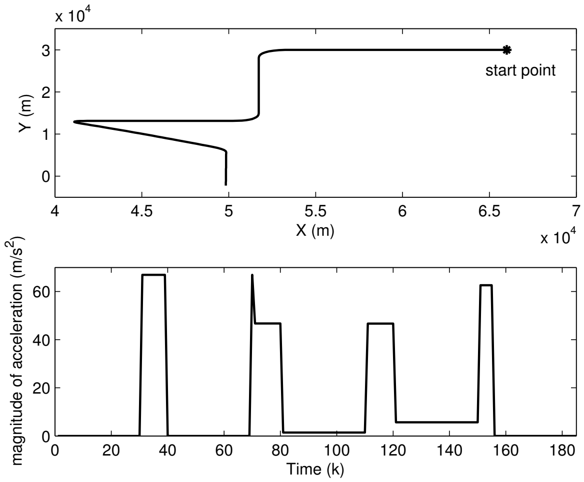


Fig. 3. Trajectory and acceleration magnitude of a target in Scenario 1

The parameters used in the simulation are:

$$\bar{\tau} = \tau_M + \frac{a_{\max} - a_k}{a_{\max}} \left(\bar{\tau}_0 - \bar{\tau}_M\right), \, \sigma_\tau = \frac{1}{12}\bar{\tau}_a$$
$$\bar{\tau}_M = 10, \, \bar{\tau}_0 = 30, \, a_{\max} = 80,$$
$$\sigma_\theta = \frac{\pi}{12}, \, P_M = 0.1, \, P_0 = \begin{cases} 0.6 & a_k \neq a_{\max} \\ 0.8 & a_k = a_{\max} \end{cases}$$

The random sojourn time τ is rounded to its nearest integer and the initial acceleration is set to zero.

B. Simulation results and discussion

The other design parameters of the estimators in the simulations, such as the transition probability matrix and the initial model probability, are not listed here but can be found in [15]. FSMM denotes the estimator with 13 fixed models, $HG_{part}MM$, $HG_{minor}MM$ stand for HGMM algorithm using the partitioned moment matching and minor model set, respectively. The measures of performance examined are position root-mean-square errors (RMSE), velocity RMSE and mode/accerleration RMSE.

Results over 1000 Monte Carlo runs of deterministic scenario and random scenario are presented in Fig. 4. It is shown that overall HGMM outperforms the FSMM algorithm. Especially, when the target maneuvers, HGMM estimators provide significantly better accuracy and a lower peak error than FSMM.

For both scenarios, we also can see from Fig. 4 that the RMS position errors for $HG_{part}MM$ are similar to those for $HG_{minor}MM$, but the RMS velocity and mode/acceleration errors of $HG_{minor}MM$ are better than those of $HG_{part}MM$, since HG_{minor} is more spread out.

Since these three algorithms use comparable numbers of models, they have similar computational complexity.

VI. Conclusions

We have presented the HGMM approach to state estimation for a hybrid system with Markovian switching parameters which belong to a continuous space. As a further investigation of [16], we also have provided some practical HGMM algorithms combined with our proposed model set design methods by moment matching. These algorithms have been adopted in the simulation of maneuvering target tracking under different scenarios, and their performance has been assessed. Results demonstrate that the HGMM estimator performs better than the corresponding FSMM estimator with a comparable computational complexity. Moreover, $HG_{minor}MM$ is more cost-effective than the others, and is recommended.

Some theoretical analyses on this HGMM are under investigation.

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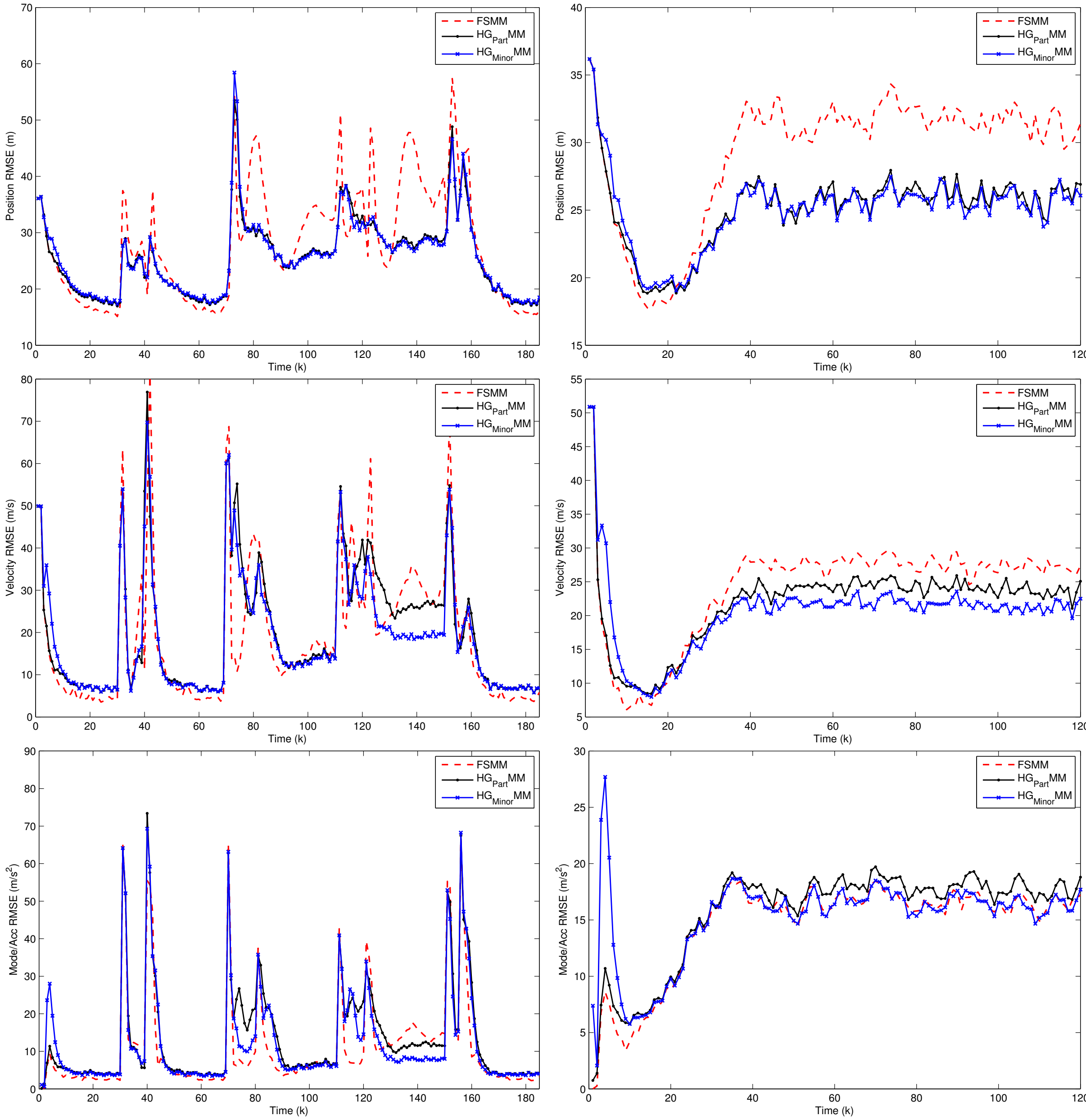

(a) under Scenario 1 (Deterministic) (b) under Scenario 2 (Random)

Fig. 4. RMS position, velocity and acceleration (mode) errors

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